

# Day 8: Unsupervised Text Scaling

Kenneth Benoit

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# Unsupervised methods scale **distance**

- ▶ Text gets converted into a quantitative matrix of **features**
  - ▶ words, typically
  - ▶ could be dictionary entries, or parts of speech
- ▶ Language is irrelevant
- ▶ Different possible definitions of *distance*
- ▶ Works on any quantitative matrix of features

# Parametric v. non-parametric methods

- ▶ **Parametric methods** model feature occurrence according to some stochastic distribution, typically in the form of a measurement model
  - ▶ for instance, model words as a multi-level Bernoulli distribution, or a Poisson distribution
  - ▶ word effects and “positional” effects are unobserved parameters to be estimated
- ▶ **Non-parametric methods** typically based on the Singular Value Decomposition of a matrix
  - ▶ correspondence analysis
  - ▶ factor analysis
  - ▶ other (multi)dimensional scaling methods

## Parametric scaling model: Model counts as Poisson

- ▶ Many dependent variables of interest may be in the form of counts of discrete events— examples:
  - ▶ international wars or conflict events
  - ▶ traffic incidents
  - ▶ deaths
  - ▶ word count given an underlying orientation
- ▶ Characteristics: these  $Y$  are bounded between  $(0, \infty)$  and take on only discrete values  $0, 1, 2, \dots, \infty$
- ▶ Imagine a social system that produces events randomly during a fixed period, and at the end of this period only the total count is observed. For  $N$  periods, we have  $y_1, y_2, \dots, y_N$  observed counts

## Poisson data model first principles

1. The probability that two events occur at precisely the same time is zero
2. During each period  $i$ , the event rate occurrence  $\lambda_i$  remains constant and is independent of all previous events during the period
  - ▶ note that this implies no *contagion* effects
  - ▶ also known as *Markov independence*
3. Zero events are recorded at the start of the period
4. All observation intervals are equal over  $i$

## The Poisson distribution

$$f_{\text{Poisson}}(y_i|\lambda) = \begin{cases} \frac{e^{-\lambda} \lambda^{y_i}}{y_i!} & \forall \lambda > 0 \text{ and } y_i = 0, 1, 2, \dots \\ 0 & \text{otherwise} \end{cases}$$

$$\text{Pr}(Y|\lambda) = \prod_{i=1}^n \frac{e^{-\lambda} \lambda^{y_i}}{y_i!}$$

$$\lambda = e^{\mathbf{X}_i \beta}$$

$$\text{E}(y_i) = \lambda$$

$$\text{Var}(y_i) = \lambda$$

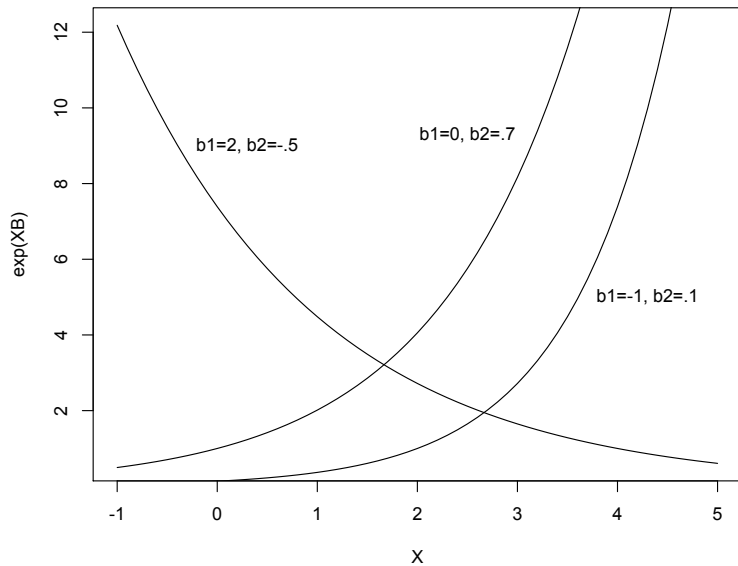
## Systematic component

- ▶  $\lambda_i > 0$  is only bounded from below (unlike  $\pi_i$ )
- ▶ This implies that the effect cannot be linear
- ▶ Hence for the functional form we will use an **exponential transformation**

$$E(Y_i) = \lambda_i = e^{X_i\beta}$$

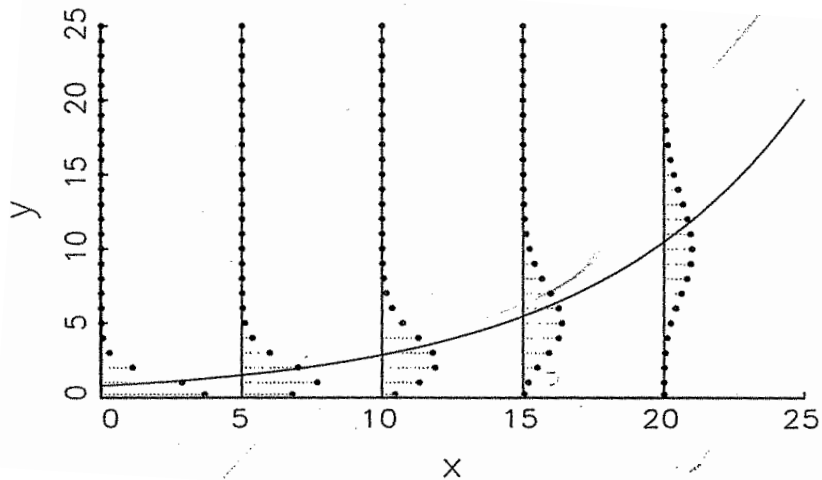
- ▶ Other possibilities exist, but this is by far the most common – indeed almost universally used – functional form for event count models

# Exponential link function





## Exponential link function



## Likelihood for Poisson

$$\begin{aligned}L(\lambda|y) &= \prod_{i=1}^N \frac{e^{-\lambda_i} \lambda_i^{y_i}}{y_i!} \\ \ln L(\lambda|y) &= \sum_{i=1}^N \ln \left[ \frac{e^{-\lambda_i} \lambda_i^{y_i}}{y_i!} \right] \\ &= \sum_{i=1}^N \left\{ \ln e^{-\lambda_i} + \ln(\lambda_i^{y_i}) + \ln \left( \frac{1}{y_i!} \right) \right\} \\ &= \sum_{i=1}^N \{-\lambda_i + y_i \ln(\lambda_i) - \ln(y_i!)\} \\ &= \sum_{i=1}^N \{-e^{X_i \beta} + y_i \ln e^{X_i \beta} - \ln y_i!\} \\ &\propto \sum_{i=1}^N \{-e^{X_i \beta} + y_i X_i \beta - \text{dropped}\} \\ \ln L(\beta|y) &\propto \sum_{i=1}^N \{X_i \beta y_i - e^{X_i \beta}\}\end{aligned}$$

# The Poisson scaling “wordfish” model

## Data:

- ▶  $Y$  is  $N$  (speaker)  $\times$   $V$  (word) term document matrix  
 $V \gg N$

## Model:

$$P(Y_i | \theta) = \prod_{j=1}^V P(Y_{ij} | \theta_i)$$
$$Y_{ij} \sim \text{Poisson}(\lambda_{ij}) \quad (\text{POIS})$$
$$\log \lambda_{ij} = (g +) \alpha_i + \theta_i \beta_j + \psi_j$$

## Estimation:

- ▶ Easy to fit for large  $V$  ( $V$  Poisson regressions with  $\alpha$  offsets)

## Model components and notation

$$\log \lambda_{ij} = \alpha_i + \theta_i \beta_j + \psi_j$$

<i>Element</i>	<i>Meaning</i>
$i$	indexes the targets of interest (political actors)
$j$	indexes word types
$\theta_i$	the unobservable political position of actor $i$
$\beta_j$	word parameters on $\theta$ – the “ideological” direction of word $j$
$\psi_j$	word “fixed effect” (function of the frequency of word $j$ )
$\alpha_i$	actor “fixed effects” (a function of (log) document length to allow estimation in Poisson of an essentially multinomial process)

## How to estimate this model

Maximum likelihood estimation using (a form of) Expectation Maximization:

- ▶ If we knew  $\Psi$  and  $\beta$  (the word parameters) then we have a Poisson regression model
- ▶ If we knew  $\alpha$  and  $\theta$  (the party / politician / document parameters) then we have a Poisson regression model too!
- ▶ So we alternate them and hope to converge to reasonable estimates for both

# The iterative (conditional) maximum likelihood estimation

Start by *guessing* the parameters

Algorithm:

- ▶ Assume the current party parameters are correct and fit as a Poisson regression model
- ▶ Assume the current word parameters are correct and fit as a Poisson regression model
- ▶ Normalize  $\theta$ s to mean 0 and variance 1

Repeat

## Identification

The *scale* and *direction* of  $\theta$  is undetermined — like most models with latent variables

To identify the model in Wordfish

- ▶ Fix one  $\alpha$  to zero to specify the left-right direction (Wordfish option 1)
- ▶ Fix the  $\hat{\theta}$ s to mean 0 and variance 1 to specify the scale (Wordfish option 2)
- ▶ Fix two  $\hat{\theta}$ s to specify the direction and scale (Wordfish option 3 and Wordscores)

Implication: Fixing two reference scores does not specify the policy domain, it just identifies the model!

## “Features” of the parametric scaling approach

- ▶ Standard (statistical) **inference** about parameters
- ▶ **Uncertainty** accounting for parameters
- ▶ **Distributional assumptions** are laid bare for inspection
  - ▶ *conditional independence*
  - ▶ *stochastic process* (e.g.  $E(Y_{ij}) = \text{Var}(Y_{ij}) = \lambda_{ij}$ )
- ▶ Permits **hierarchical reparameterization** (to add covariates)
- ▶ Prediction: in particular, **out of sample prediction**



## Assumptions of the model

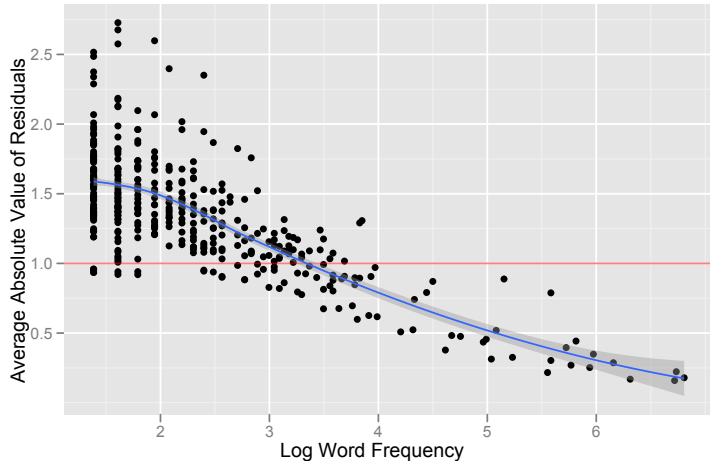
- ▶ Words occur in order  
In occur words order.  
Occur order words in.  
“No more training do you require. Already know you that which you need.” (Yoda)
- ▶ Words occur in combinations  
“carbon tax” / “income tax” / “inheritance tax” / “capital gains tax” / “bank tax”
- ▶ Sentences (and topics) occur in sequence (extreme serial correlation)
- ▶ Style may mean means we are likely to use synonyms – very probable. In fact it’s very distinctly possible, to be expected, odds-on, plausible, imaginable; expected, anticipated, predictable, predicted, foreseeable.)
- ▶ Rhetoric may lead to repetition. (“Yes we can!”) – anaphora

## Assumptions of the model (cont.)

- ▶ Poisson assumes  $\text{Var}(Y_{ij}) = E(Y_{ij}) = \lambda_{ij}$
- ▶ For many reasons, we are likely to encounter overdispersion or underdispersion
  - ▶ **over**dispersion when “informative” words tend to cluster together
  - ▶ **under**dispersion could (possibly) occur when words of high frequency are uninformative and have relatively low between-text variation (once length is considered)
- ▶ This should be a *word*-level parameter

# Overdispersion in German manifesto data

(from Slapin and Proksch 2008)



# Non-parametric methods

- ▶ Non-parametric methods are algorithmic, involving no “parameters” in the procedure that are estimated
- ▶ Hence there is no uncertainty accounting given distributional theory
- ▶ Advantage: don't have to make assumptions
- ▶ Disadvantages:
  - ▶ cannot leverage probability conclusions given distributional assumptions and statistical theory
  - ▶ results highly fit to the data
  - ▶ not really assumption-free, if we are honest

# Correspondence Analysis

- ▶ CA is like factor analysis for categorical data
- ▶ Following normalization of the marginals, it uses Singular Value Decomposition to reduce the dimensionality of the word-by-text matrix
- ▶ This allows projection of the positioning of the words as well as the texts into multi-dimensional space
- ▶ The number of dimensions – as in factor analysis – can be decided based on the eigenvalues from the SVD

# Singular Value Decomposition

- ▶ A matrix  $\mathbf{X}$  can be represented in a dimensionality equal to its rank  $k$  as:

$$\mathbf{X} = \mathbf{U} \mathbf{d} \mathbf{V}' \quad (1)$$

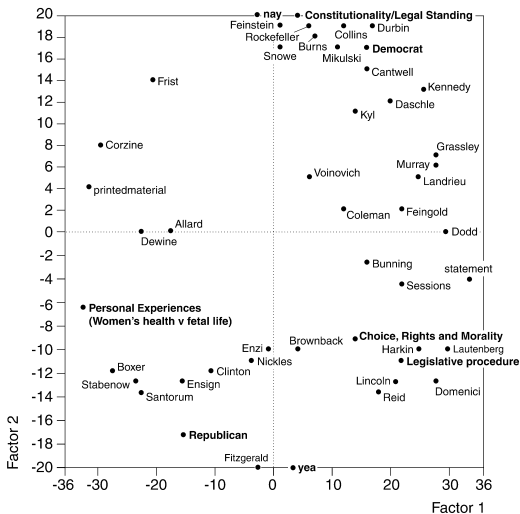
$i \times j$        $i \times k$     $k \times k$     $j \times k$

- ▶ The  $\mathbf{U}$ ,  $\mathbf{d}$ , and  $\mathbf{V}$  matrixes “relocate” the elements of  $\mathbf{X}$  onto new coordinate vectors in  $n$ -dimensional Euclidean space
- ▶ Row variables of  $\mathbf{X}$  become points on the  $\mathbf{U}$  column coordinates, and the column variables of  $\mathbf{X}$  become points on the  $\mathbf{V}$  column coordinates
- ▶ The coordinate vectors are perpendicular (*orthogonal*) to each other and are normalized to unit length

# Correspondence Analysis and SVD

- ▶ Divide each value of  $\mathbf{X}$  by the geometric mean of the corresponding marginal totals (square root of the product of row and column totals for each cell)
  - ▶ Conceptually similar to subtracting out the  $\chi^2$  expected cell values from the observed cell values
- ▶ Perform an SVD on this transformed matrix
  - ▶ This yields singular values  $\mathbf{d}$  (with first always 1.0)
- ▶ Rescale the row ( $\mathbf{U}$ ) and column ( $\mathbf{V}$ ) vectors to obtain canonical scores (rescaled as  $U_i\sqrt{f_{..}/f_{i.}}$  and  $V_j\sqrt{f_{..}/f_{.j}}$ )

# Example: Schonhardt-Bailey (2008) - speakers

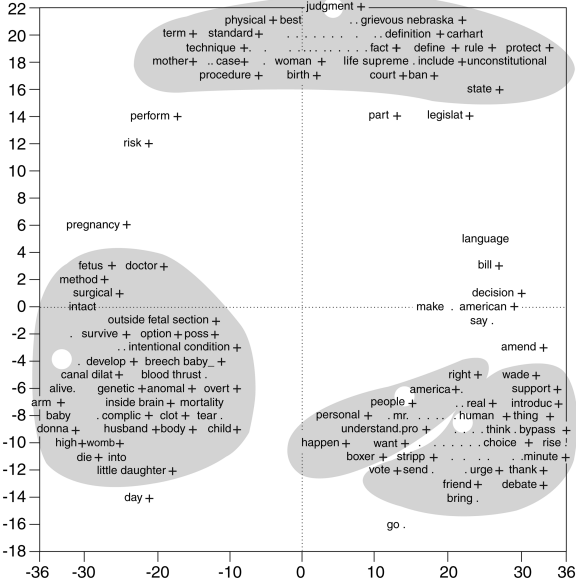


	Eigenvalue	% Association	% Cumulative
Factor 1	0.30	44.4	44.4
Factor 2	0.22	32.9	77.3

Fig. 3 Correspondence analysis of classes and tags from Senate debates on Partial-Birth Abortion Ban Act



# Example: Schonhardt-Bailey (2008) - words



# How to get confidence intervals for CA

- ▶ There are problems with bootstrapping: (Milan and Whittaker 2004)
  - ▶ rotation of the principal components
  - ▶ inversion of singular values
  - ▶ reflection in an axis

## How to account for uncertainty?

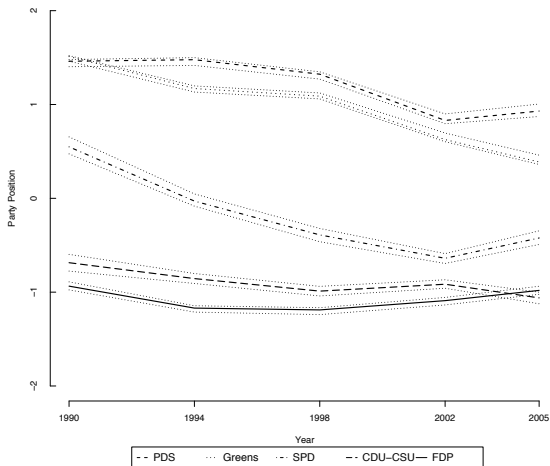
- ▶ Don't. (SVD-like methods, e.g. correspondence analysis)
- ▶ Analytical derivatives
- ▶ Parametric bootstrapping (Slapin and Proksch, Lewis and Poole)
- ▶ Non-parametric bootstrapping
- ▶ (and yes of course) Posterior sampling from MCMC

## Methods of uncertainty accounting in text scaling

	MCMC	Conditional ML	SVD-based	Algorithmic
Uncertainty accounting	(multinomial+)	(Poisson)	(CA)	(Wordscores)
Posterior sampling	✓			
Analytical		✓	??	?
Parametric bootstrap		✓		
Non-parametric BS		✓	?	✓

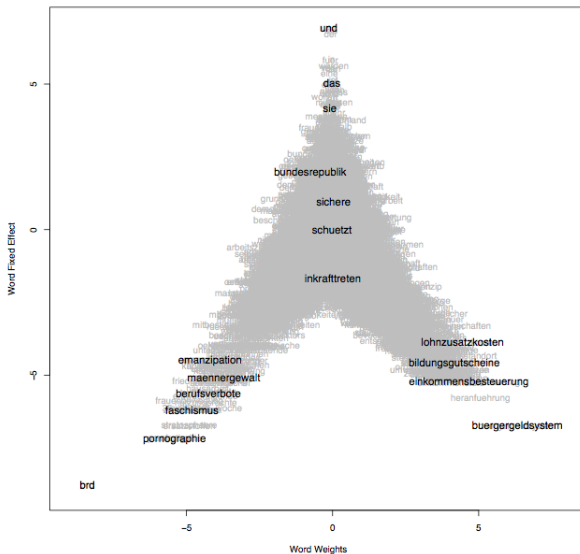
# Parametric Bootstrapping and analytical derivatives yield “errors” that are too small

Left-Right Positions in Germany, 1990–2005  
including 95% confidence intervals



# Frequency and informativeness

$\Psi$  and  $\beta$  (frequency and informativeness) tend to trade-off





# Dimensions

How infer more than one dimension?

This is two questions:

- ▶ How to get two dimensions (for all policy areas) at the same time?
- ▶ How to get one dimension for each policy area?



## Interpreting multiple dimensions

To get one dimension for each policy area, split up the document by hand and use the subparts as documents (the Slapin and Proksch method)

There is currently *no* implementation of Wordscores or Wordfish that extracts two or more dimensions at once

- ▶ But since Wordfish is a type of factor analysis model, there is no reason in principle why it could not

## Interpreting scaled dimensions

- ▶ How to interpret  $\hat{\theta}$ s substantively? *assert...*
- ▶ Billy Joe Jimbob's "Make a Wish Foundation"



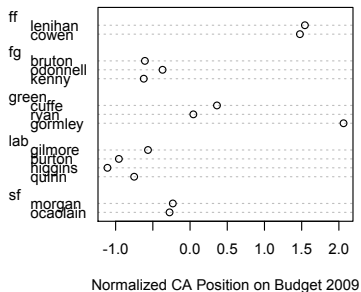
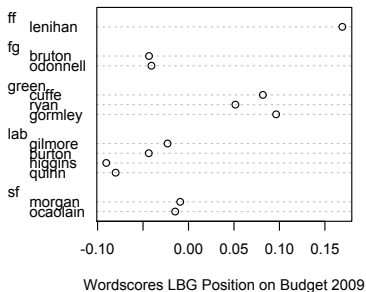
(Billy Joe Jimbob)



## Interpreting scaled dimensions

- ▶ Another (better) option: compare them other known descriptive variables
- ▶ Hopefully also *validate* the scale results with some human judgments
- ▶ This is necessary even for single-dimensional scaling
- ▶ And just as applicable for non-parametric methods (e.g. correspondence analysis) as for the Poisson scaling model

# What happens if we include irrelevant text?



## What happens if we include irrelevant text?



John Gormley: leader of the Green Party and Minister for the Environment, Heritage and Local Government

“As leader of the Green Party I want to take this opportunity to set out my party’s position on budget 2010. . .”

[772 words later]

“I will now comment on some specific aspects of my Department’s Estimate. I will concentrate on the principal sectors within the Department’s very broad remit . . .”

# Without irrelevant text

